LESSON 7.3

Products, Quotients, and Roots of Monomials

CA Standards AF 2.2

Goal: • Apply properties of exponents to monomials.

Vocabulary

Monomial: A number, a variable, or a product of a number and one or more variables with whole number exponents

EXAMPLE 1 Finding Products of Monomials

a. $(-4x)(3x^5) = \boxed{-4} \cdot x \cdot \boxed{3} \cdot \boxed{x^5}$ Expand the expression.

$= \boxed{-4} \cdot \boxed{3} \cdot x \cdot \boxed{x^5}$ Regroup factors.

$= \boxed{-12} \cdot x \cdot \boxed{x^5}$ Multiply coefficients.

$= \boxed{-12\,x^6}$ Product of powers property

b. $(6ab^4)^2 = 6^2 \cdot \boxed{a^2} \cdot (\boxed{b^4})^{\boxed{2}}$ Power of a product property

$= \boxed{36} \cdot \boxed{a^2} \cdot (\boxed{b^4})^{\boxed{2}}$ Evaluate power of coefficient.

$= \boxed{36a^2b^8}$ Power of a power property

EXAMPLE 2 Finding Quotients of Monomials

a. $\frac{10a^5b^2}{2a^3b^4} = \frac{10}{2} \cdot \boxed{\frac{a^5}{a^3}} \cdot \boxed{\frac{b^2}{b^4}}$ Expand the expression.

$= \boxed{5} \cdot \frac{a^5}{a^3} \cdot \frac{b^2}{b^4}$ Divide coefficients.

$= 5 \cdot \boxed{a^2} \cdot \boxed{b^{-2}}$ Quotient of powers property

$= \boxed{\frac{5a^2}{b^2}}$ Definition of negative exponent

b. $\left(\frac{p^4}{q}\right)^3 = \boxed{\frac{(p^4)^3}{q^3}}$ Power of a quotient property

$= \boxed{\frac{p^{12}}{q^3}}$ Power of a power property

Guided Practice **Simplify the expression.**

1. $(5n^3)(-4n^2)$	**2.** $(3ab^4)(9a^2b^5)$	**3.** $(4x^2y)^3$
4. $\frac{2x^{10}}{6x^3}$	**5.** $\frac{9y^4z}{9yz^2}$	**6.** $\frac{5d^3}{f^3}$

EXAMPLE 3 Using Properties of Exponents

Geometry The volume V of a cylindrical can with a height of 4 inches and radius r inches is given by $V = 4\pi r^2$.

Solution

Use a ratio to compare the volume of a can with radius $2r$ inches with the volume of a can with radius r inches.

$$\frac{\text{Radius } 2r \text{ inches}}{\text{Radius } r \text{ inches}} = \frac{4\pi\boxed{(2r)}^2}{4\pi\boxed{r^2}}$$ Write ratio.

$$= \frac{4\pi \cdot \boxed{2^2} \cdot \boxed{r^2}}{4\boxed{\pi r^2}}$$ Power of a product property

$$= \frac{\boxed{16\pi r^2}}{4\boxed{\pi r^2}}$$ Simplify numerator.

$$= 4 \cdot \frac{\pi}{\pi} \cdot \boxed{\frac{r^2}{r^2}}$$ Divide coefficients.

$$= \boxed{4}$$ Quotient of powers property

Answer When the radius is doubled, the volume of the can is $\boxed{4}$ times as great.

In this book, all variables in radical expressions represent nonnegative numbers.

SQUARE ROOTS OF SQUARED EXPRESSIONS

Words If a is a nonnegative number or expression, then the $\boxed{\text{square root}}$ of a squared is $\boxed{a}$.

Algebra $\sqrt{a^2} = \boxed{a}$, $a \geq 0$ **Numbers** $\sqrt{4^2} = \boxed{4}$

EXAMPLE 4 Simplifying Square Roots

a. $\sqrt{36z^2} = \boxed{\sqrt{6^2 \cdot z^2}}$ Write 36 as $\boxed{6}^2$.

$= \boxed{\sqrt{6^2}} \cdot \boxed{\sqrt{z^2}}$ Product property of square roots

$= \boxed{6z}$ Simplify square roots of squared expressions.

b. $\sqrt{32m^2} = \boxed{\sqrt{4^2 \cdot 2 \cdot m^2}}$ Factor greatest perfect square from 32.

$= \boxed{\sqrt{4^2}} \cdot \boxed{\sqrt{2^2}} \cdot \boxed{\sqrt{m^2}}$ Product property of square roots

$= \boxed{4} \cdot \boxed{\sqrt{2}} \cdot \boxed{m}$ Simplify square roots of squared expressions.

$= \boxed{4m\sqrt{2}}$ Commutative property

In part (b), the final expression is written as $4m\sqrt{2}$ rather than as $4\sqrt{2}\,m$ to make it clear that the variable is not under the radical sign.

c. $\sqrt{a^4b^8} = \boxed{\sqrt{(a^2)^2 \cdot (b^4)^2}}$ Write a^4 and b^8 as squares of expressions.

$= \boxed{\sqrt{(a^2)^2}} \cdot \boxed{\sqrt{(b^4)^2}}$ Product property of square roots

$= \boxed{a^2b^4}$ Simplify square roots of squared expressions

Guided Practice Simplify the expression.

Homework

7. $\sqrt{25x^2}$	**8.** $\sqrt{18k^2}$	**9.** $\sqrt{c^6d^2}$