Exponent Properties Involving Quotients

Goal: Use exponent properties involving quotients.

Quotient of Powers Property

Words To divide two powers with the same nonzero $\boxed{\text{base}}$, $\boxed{\text{subtract}}$ the exponent of the denominator from the exponent of the numerator.

Algebra $\dfrac{a^m}{\boxed{a^n}} = a^{\boxed{m-n}}, a \neq 0$ **Numbers** $\dfrac{6^8}{6^6} = 6^{\boxed{8-6}} = \boxed{6^2}$

EXAMPLE 1 **Using the Quotient of Powers Property**

Simplify the expression. Write your answer as a power.

a. $\dfrac{x^{11}}{x^3} = x^{\boxed{11-3}}$ Quotient of powers property

$= \boxed{x^8}$ Subtract exponents.

b. $\dfrac{7^9}{7^6} = 7^{\boxed{9-6}}$ Quotient of powers property

$= \boxed{7^3}$ Subtract exponents.

c. $\dfrac{y^{-7}}{y^{-12}} = y^{\boxed{-7-(-12)}}$ Quotient of powers property

$= y^{\boxed{5}}$ Subtract exponents.

WATCH OUT !

When using the quotient of powers property, always subtract the exponent of the denominator from the exponent of the numerator.

POWER OF A QUOTIENT PROPERTY

Words To find the power of a quotient, find the power of the $\boxed{\text{numerator}}$ and the power of the $\boxed{\text{denominator}}$, then $\boxed{\text{divide}}$.

Algebra $\left(\dfrac{a}{b}\right)^m = \boxed{\dfrac{a^m}{b^m}}, b \neq 0$ **Words** $\left(\dfrac{3}{4}\right)^5 = \boxed{\dfrac{3^5}{4^5}}$

EXAMPLE **2** **Using the Power of a Quotient Property**

a. $\left(\dfrac{2}{-9}\right)^2 = \dfrac{2^{\boxed{2}}}{(-9)^{\boxed{2}}}$ Power of a quotient property

$= \boxed{\dfrac{4}{81}}$ Evaluate powers.

b. $\left(\dfrac{6}{d}\right)^3 = \dfrac{6^{\boxed{3}}}{d^{\boxed{3}}}$ Power of a quotient property

$= \boxed{\dfrac{216}{d^3}}$ Evaluate power.

Need help with negative exponents? see page 206 of your textbook

c. $\left(\dfrac{z}{4}\right)^{-4} = \dfrac{z^{\boxed{-4}}}{4^{\boxed{-4}}}$ Power of a quotient property

$= \dfrac{\boxed{\dfrac{1}{z^4}}}{\boxed{\dfrac{1}{4^4}}}$ Definition of negative exponents

$= \dfrac{1}{z^4} \bullet \boxed{\dfrac{4^4}{1}}$ Multiply by reciprocal.

$= \boxed{\dfrac{256}{z^4}}$ Evaluate power and simplify.

Guided Practice **Simplify the expression.**

1. $\dfrac{10^5}{10^4}$	2. $\dfrac{a^8}{a^4}$	3. $\dfrac{3^{-2}}{3^{-6}}$
4. $\left(\dfrac{5}{8}\right)^2$	5. $\left(\dfrac{b}{2}\right)^5$	6. $\left(\dfrac{7}{c}\right)^{-3}$

EXAMPLE 3 **Dividing Numbers in Scientific Notation**

Population The population of Los Angeles, California in 2004 was about 3.85×10^6. The population of Bridgeport, Connecticut in 2004 was about 1.40×10^5. The population of Los Angeles was about how many times the population of Bridgeport?

Solution

Find the ratio of the population of Los Angeles to the population of Bridgeport.

$$\frac{3.85 \times 10^6}{1.40 \times 10^5} = \frac{\boxed{3.85}}{\boxed{1.40}} \times \frac{\boxed{10^6}}{\boxed{10^5}} \qquad \text{Write quotient as product.}$$

$$= \boxed{2.75} \times 10^{\boxed{6-5}} \qquad \text{Quotient of powers property}$$

$$= \boxed{2.75} \times 10^{\boxed{1}} \qquad \text{Simplify exponent.}$$

$$= \boxed{27.5} \qquad \text{Write in standard form.}$$

Answer In 2004, the population of Los Angeles was $\boxed{27.5}$ times the population of Bridgeport.

Homework

PROPERTIES OF EXPONENTS

Let m and n be integers and let a and b be real numbers.

Property	Definition	Example
Negative Exponent	$a^{-m} = \frac{1}{a^m}, a \neq 0$	$2^{-5} = \boxed{\frac{1}{2^5}}$
Zero Exponent	$a^0 = \boxed{1}, a \neq 0$	$(-4)^0 = \boxed{1}$
Product of Powers	$a^m \cdot a^n = \boxed{a^{m+n}}$	$8^5 \cdot 8^4 = \boxed{8^9}$
Power of a Power	$(a^m)^n = \boxed{a^{mn}}$	$(3^2)^5 = \boxed{3^{10}}$
Power of a Product	$(ab)^m = \boxed{a^m \cdot b^m}$	$(6x)^2 = \boxed{6^2 x^2}$
Quotient of Powers	$\frac{a^m}{a^n} = \boxed{a^{m-n}}$	$\frac{4^9}{4^3} = \boxed{4^6}$
Power of a Quotient	$\left(\frac{a}{b}\right)^m = \boxed{\frac{a^m}{b^m}}, b \neq 0$	$\left(\frac{9}{10}\right)^3 = \boxed{\frac{9^3}{10^3}}$